

JPL

# EFFECT OF LONG PERIOD OCEAN TIDES ON THE EARTH'S ROTATION

by

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- Spectra of polar motion excitation functions exhibit enhanced power in fortnightly tidal band
  - This enhanced power is attributed to ocean tidal excitation
- Upon subtracting atmospheric wind and pressure effects, fits for periodic terms at the  $Mf$  and  $Mf'$  tidal frequencies are obtained and compared to predictions of ocean tide models
  - Ocean tide models predict polar motion excitation effects that differ with each other, and with observations, by factors as large as 2-3
- Need improved models for effect of long-period ocean tides on Earth's rotation

# APPROACH

- **Polar motion excitation function**
  - Use that derived from SPACE95 polar motion values
    - SPACE95 is a Kalman filter-based combination of space-geodetic Earth rotation measurements
  - Spans 1976.8–1995 at 1-day intervals
- **Remove effects of atmospheric wind and pressure**
  - Use the NCEP/NCAR reanalysis atmospheric angular momentum values
    - Spans 1979–1995 at 6-hour intervals
  - Pressure term used is that computed assuming oceans respond as inverted barometer to imposed atmospheric pressure changes
- **Least-squares fit for periodic terms at tidal frequencies to SPACE95-AAM residual series**
  - Fit for mean, trend, and periodic terms at the semiannual ( $M_2$  and  $M_2'$ ), fortnightly ( $M_4$  and  $M_4'$ ), monthly ( $M_m$ ), semiannual ( $S_{sa}$ ), and annual ( $S_a$ ) tidal frequencies
  - Fit to entire 17-year span of SPACE95-AAM residual series
    - Data set spanning about 18.6 years must be used in order to resolve periodic terms at the  $M_2$  and  $M_2'$  tidal frequencies, or at the  $M_4$  and  $M_4'$  tidal frequencies, since that is their beat period
- **Compare recovered empirical tidal effects with those predicted by ocean tide models**
  - Seiler (1991) as analyzed for polar motion effects by Gross (1993) and Brosche & Wünsch (1994)
  - Dickman (1993) • Desai (1996)

# LONG PERIOD LIOUVILLE EQUATION

- Conservation of angular momentum expressed within rotating, body-fixed reference frame

$$\frac{\partial \mathbf{L}}{\partial t} + \boldsymbol{\omega} \times \mathbf{L} = \boldsymbol{\tau}$$

where the angular momentum vector  $\mathbf{L} = \boldsymbol{\cdot} \boldsymbol{\omega} + \mathbf{h}$

- Assume rotation is small perturbation from state of uniform rotation at rate  $\Omega$ . Keeping terms to first order yields long period Liouville eq.

$$\mathbf{m}(t) + \frac{i}{\sigma_{cw}} \frac{\partial \mathbf{m}}{\partial t} = \psi(t) = \chi(t) - \frac{i}{\Omega} \frac{\partial \chi}{\partial t}$$

where:  $\mathbf{m} \equiv \boldsymbol{\omega}_1 + i \boldsymbol{\omega}_2 / \Omega$  ('terrestrial' location of rotation pole)  
 $\psi(t), \chi(t)$  are the polar motion excitation functions  
 $\sigma_{cw}$  is complex-valued frequency of Chandler wobble

- Written in terms of reported polar motion parameters  $\mathbf{p}(t) \equiv \mathbf{x}_p(t) - i \mathbf{y}_p(t)$

• In time domain:

$$\mathbf{p}(t) + \frac{i}{\sigma_{cw}} \frac{\partial \mathbf{p}}{\partial t} = \chi(t) = \frac{1.61}{\Omega(C-A)} \left[ \mathbf{h}(t) + \frac{\Omega \mathbf{c}(t)}{1.44} \right]$$

• In frequency domain:

$$\mathbf{p}(\sigma) = \frac{\sigma_{cw}}{\sigma_{cw} - \sigma} \chi(\sigma)$$

# POLAR MOTION EXCITATION FUNCTIONS

- In Earth rotation theory, the excitation functions, or  $\chi$ -functions, are the forcing functions that cause changes in the Earth's rotation (length-of-day) and orientation (polar motion)
  - In general, they are functions of changes in
    - the Earth's inertia tensor
    - relative angular momentum
  - At frequencies far from the Free Core Nutation resonance (that is, at periods long compared to a day), the polar motion excitation functions  $\chi(t)$  are related to the polar motion parameters  $x_p(t)$  and  $y_p(t)$  by:
- $$\mathbf{p}(t) + \frac{i}{\sigma_{cw}} \frac{\partial \mathbf{p}}{\partial t} = x_p(t) + i y_p(t) \quad \chi(t) = \chi_1(t) + i \chi_2(t)$$
- where:  $\mathbf{p}(t) = x_p(t) - i y_p(t)$
  - This is the equation for simple harmonic motion in the complex plane
  - The excitation pole is that pole about which the rotation pole instantaneously revolves
  - Changes in the excitation pole force changes in the polar motion
  - Can be recovered from polar motion observations either by direct numerical differentiation or by deconvolution
  - The SPACE95 polar motion excitation functions have been used here

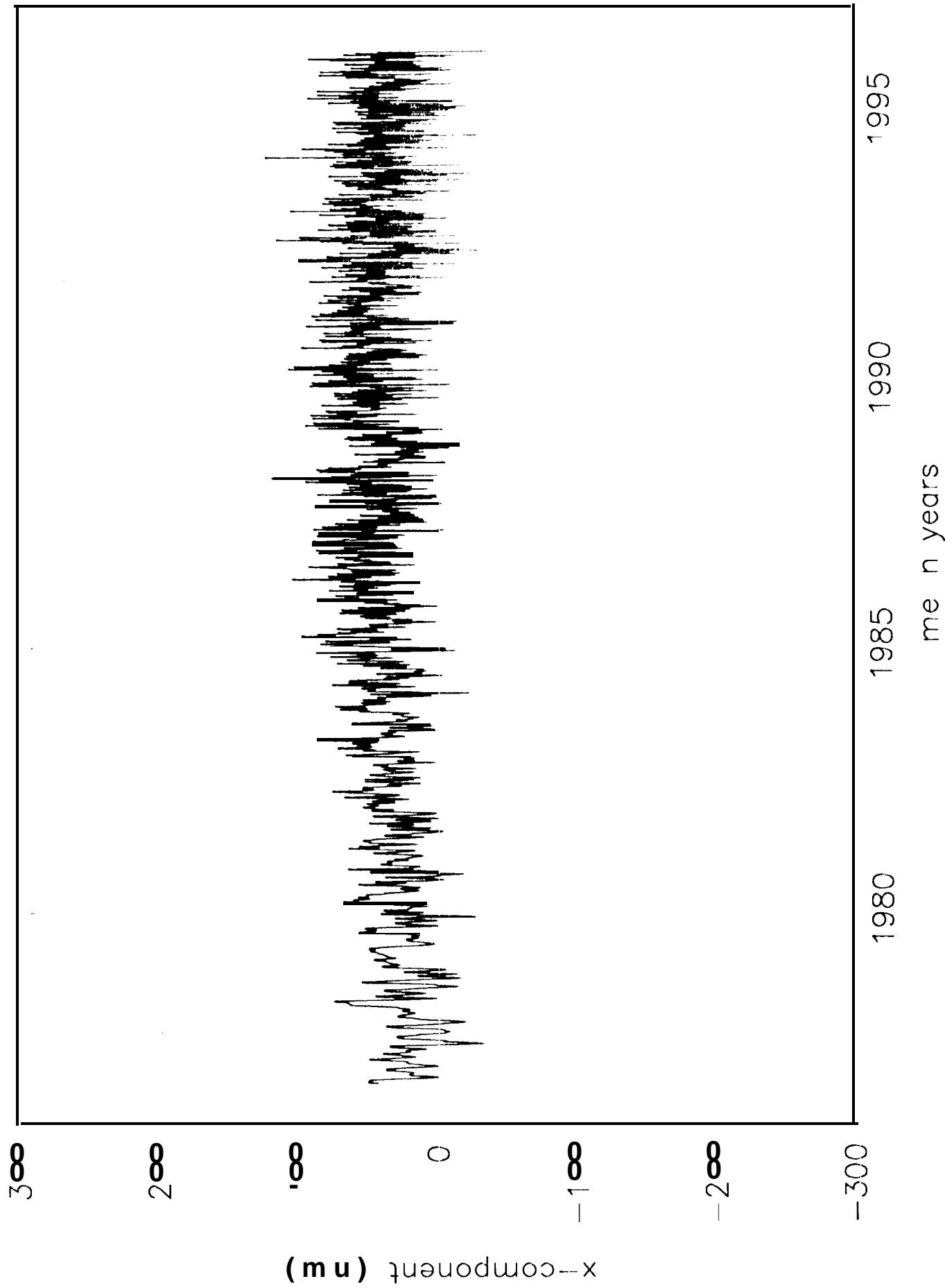
# SPACE95 EARTH ORIENTATION SERIES

- A combination of space-geodetic Earth rotation measurements
  - LLR (from JPL analysis center)
  - SLR (from University of Texas Center for Space Research analysis center)
  - VLBI (from IRIS "Intensive" (both NOAA & USNO analyses), NASA's Deep Space Network at JPL, and NASA's Space Geodesy Program at GSFC)
  - GPS (from SIC and JPL analysis centers)
- Individual series adjusted prior to their combination
  - Lead seconds and tidal terms removed (when necessary) from UT1 values
    - Yoder et al. [1981] model used to remove effect of all long period solid Earth tides
    - Dickman [1993] model used to remove ocean tidal corrections (he Yoder et al. 1981)] model values at the  $Mf$ ,  $Mf'$ ,  $Mm$ , and  $Ssa$  tidal frequencies
    - Herring [1993] empirical model used to remove effect of semidiurnal and diurnal ocean tides on NOAA's IRIS "Intensive" UT1 values
  - Bias and rate of each series adjusted to be in agreement with each other
  - Stated uncertainties of each series adjusted so its residuals with respect to a combination of all other series has a reduced chi-square of one
  - Outlying data points deleted
- Adjusted series combined using Kalman filter to form SPACE95
  - Consists of values for PMM, UT1-UTG, their "formal" uncertainties and correlations spanning October 5, 1976 to February 10, 1995 at daily intervals

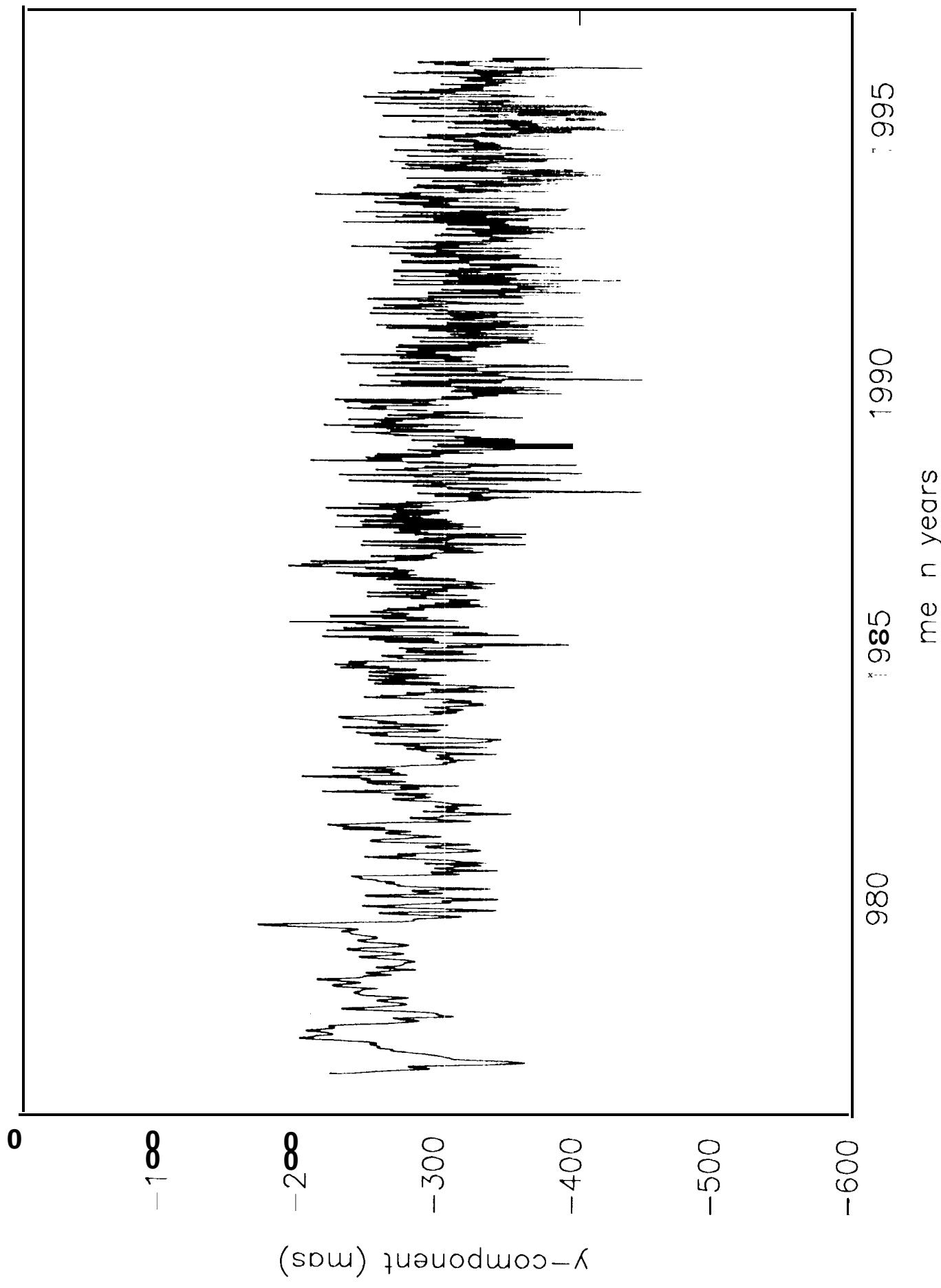
# SPACE95 POLAR MOTION EXCITATION FUNCTION

- SPACE95 consists of values for polar motion and UT1–UTC
- Kalman filter used to generate SPACE95
  - Contains a model for the polar motion process
  - Produces estimates of excitation functions as "we" as solar motion and UT1–UTC
- Polar motion excitation functions used here are those estimated by Kalman filter when generating SPACE95

# SPAC<95 POLAR MOTION EXCITATION FUNCTION



# SPAC≤95 POLAR MOTION EXCITATION FUNCTION



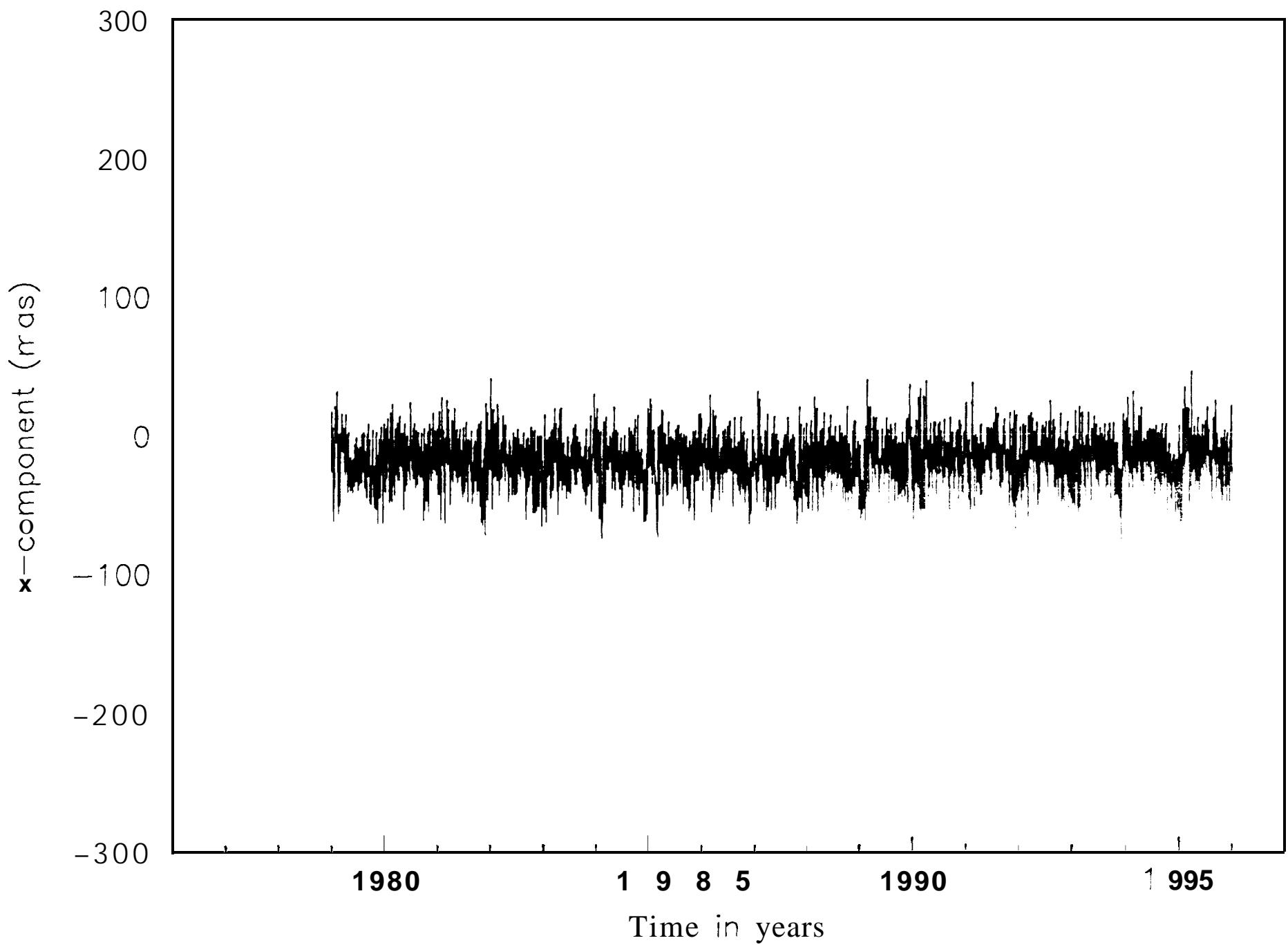
# ATMOSPHERIC ANGULAR MOMENTUM (AAM)

- Angular momentum of atmosphere changes due to:
  - Changes in strength and direction of atmospheric winds
  - Changes in mass distribution of atmosphere (changes in atmospheric pressure)
- Under principle of conservation of angular momentum, the rotation of the solid Earth changes as AAM is exchanged with the solid Earth
- AAM  $\chi$ -functions quantify the atmospheric excitation of Earth rotation
  - AAM pressure term (inertia tensor)
  - AAM wind term (relative angular momentum)
$$\chi_1^P + i \chi_2^P = \frac{-1.00 a^4}{(C-A) g} \int p_s \sin\phi \cos^2\phi (\cos\lambda + i \sin\lambda) d\lambda d\phi$$
  - AAM wind term (relative angular momentum)
$$\chi_1^W + i \chi_2^W = \frac{-1.43 a^3}{\Omega(C-A) g} \int (u \sin\phi \cos\phi + i v \cos\phi) (\cos\lambda + i \sin\lambda) dp d\lambda d\phi$$
- AAM  $\chi$ -functions are computed from the operational analyses of the:
  - Japan Meteorological Agency (JMA)
  - United Kingdom Met Office (UKMO)
  - National Centers for Environmental Prediction (NCEP)
  - European Centre for Medium-Range Weather Forecasts (ECMWF)
- AAM  $\chi$ -functions are computed from the reanalysis systems of the:
  - NASA / GSFC Data Assimilation Office (DAO)
  - NCEP / NCAR
  - ECMWF
- The NCEP / NCAR reanalysis AAM  $\chi$ -functions were used here

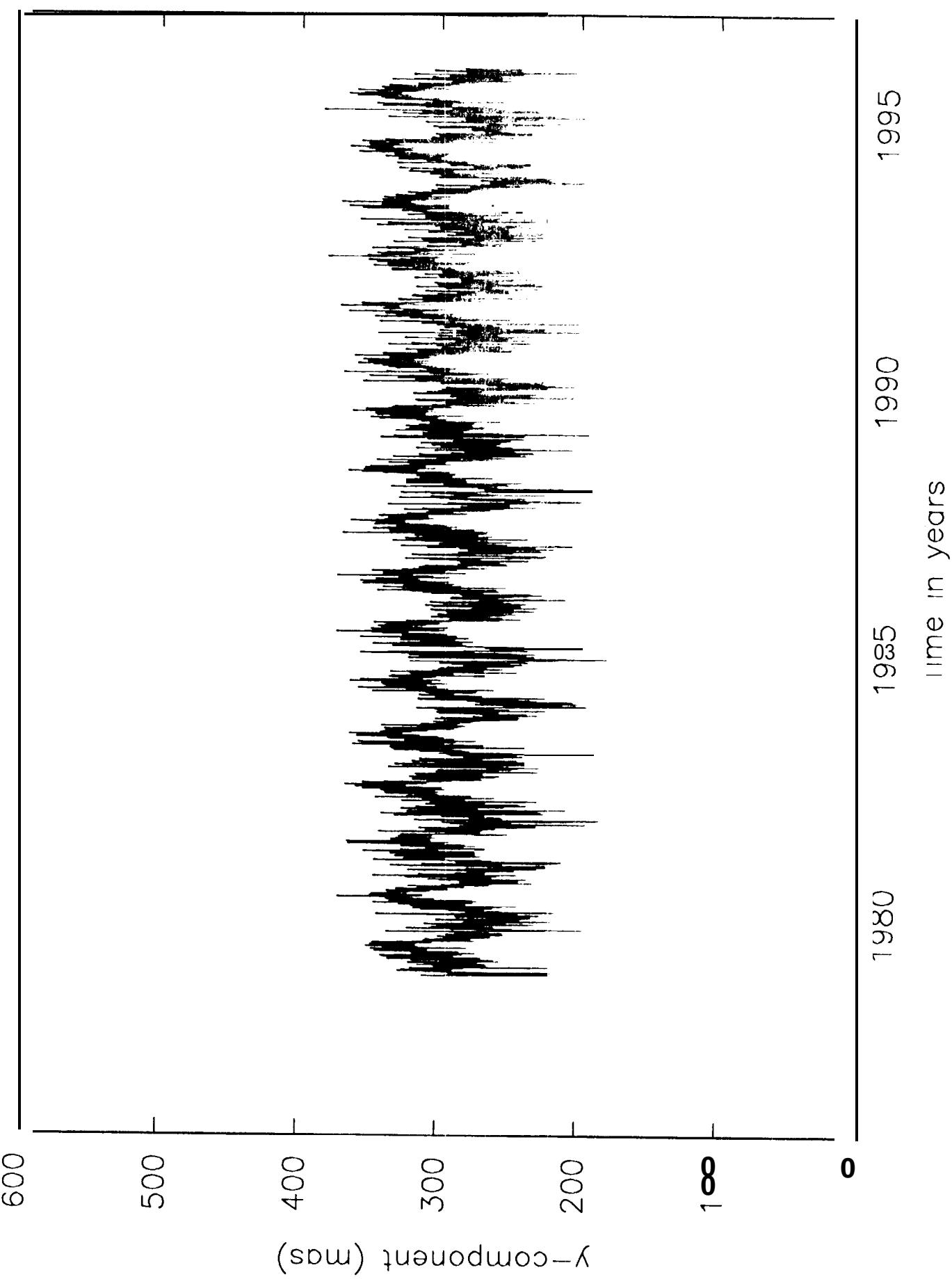
# OCEANIC RESPONSE TO ATMOSPHERIC SURFACE PRESSURE FLUCTUATIONS

- How do oceans transmit atmospheric surface pressure fluctuations to ocean bottom?
  - For AAM pressure term, need pressure evaluated at crustal surface
- Inverted barometer assumption
  - Ocean response to imposed atmospheric surface pressure fluctuations is such that pressure at ocean bottom does not change
  - Generally held to be valid at long periods ( $>$  a few days)
- Rigid ocean (no inverted barometer) assumption
  - Atmospheric surface pressure fluctuations fully transmitted (without attenuation) to the ocean bottom
- AAM pressure terms are available that have been computed under each of these assumptions
- AAM pressure term computed under inverted barometer assumption chosen for use here

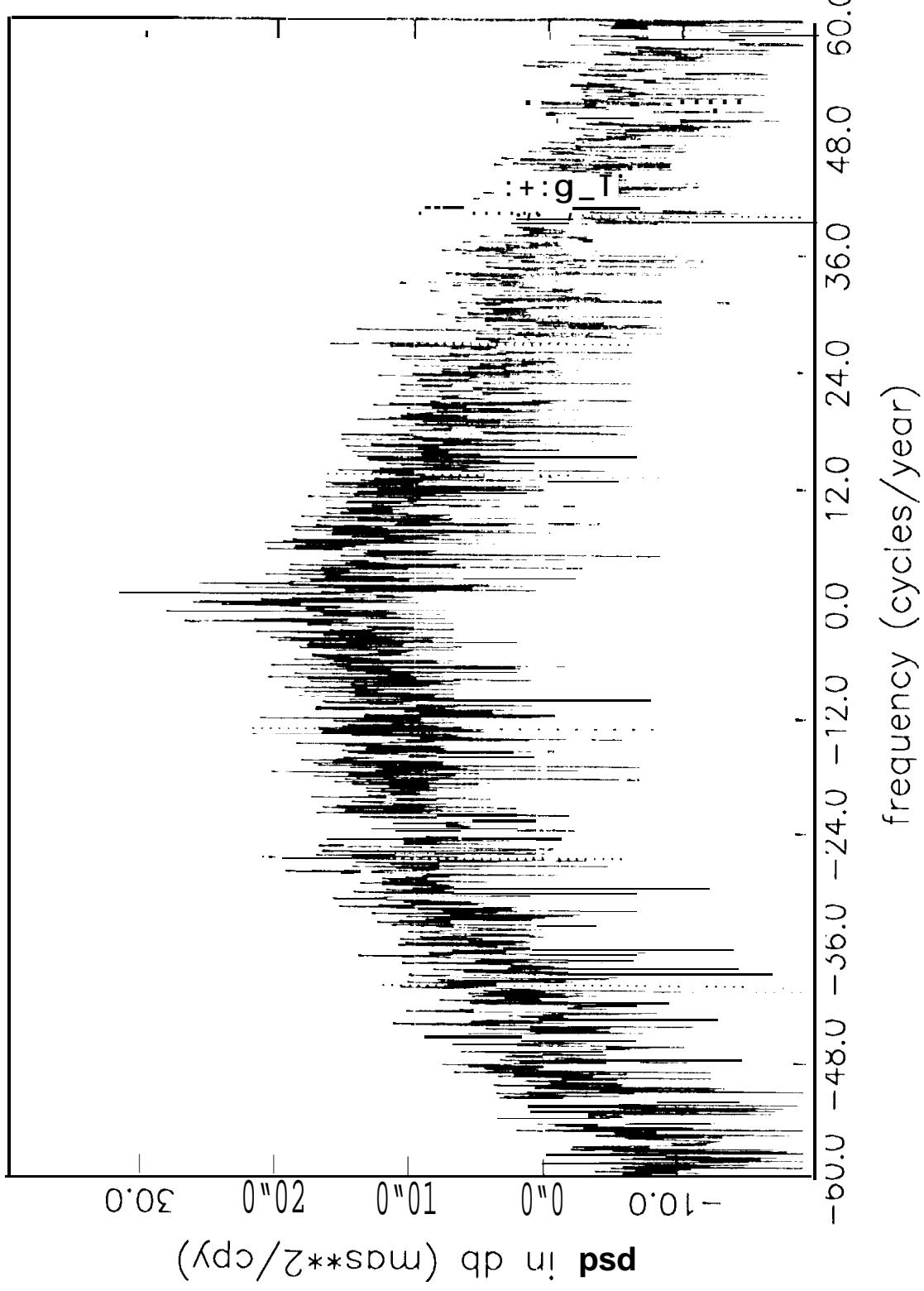
TOTAL (WIND + IB PRESSURE) AAM CW-WNC<sup>TION</sup>



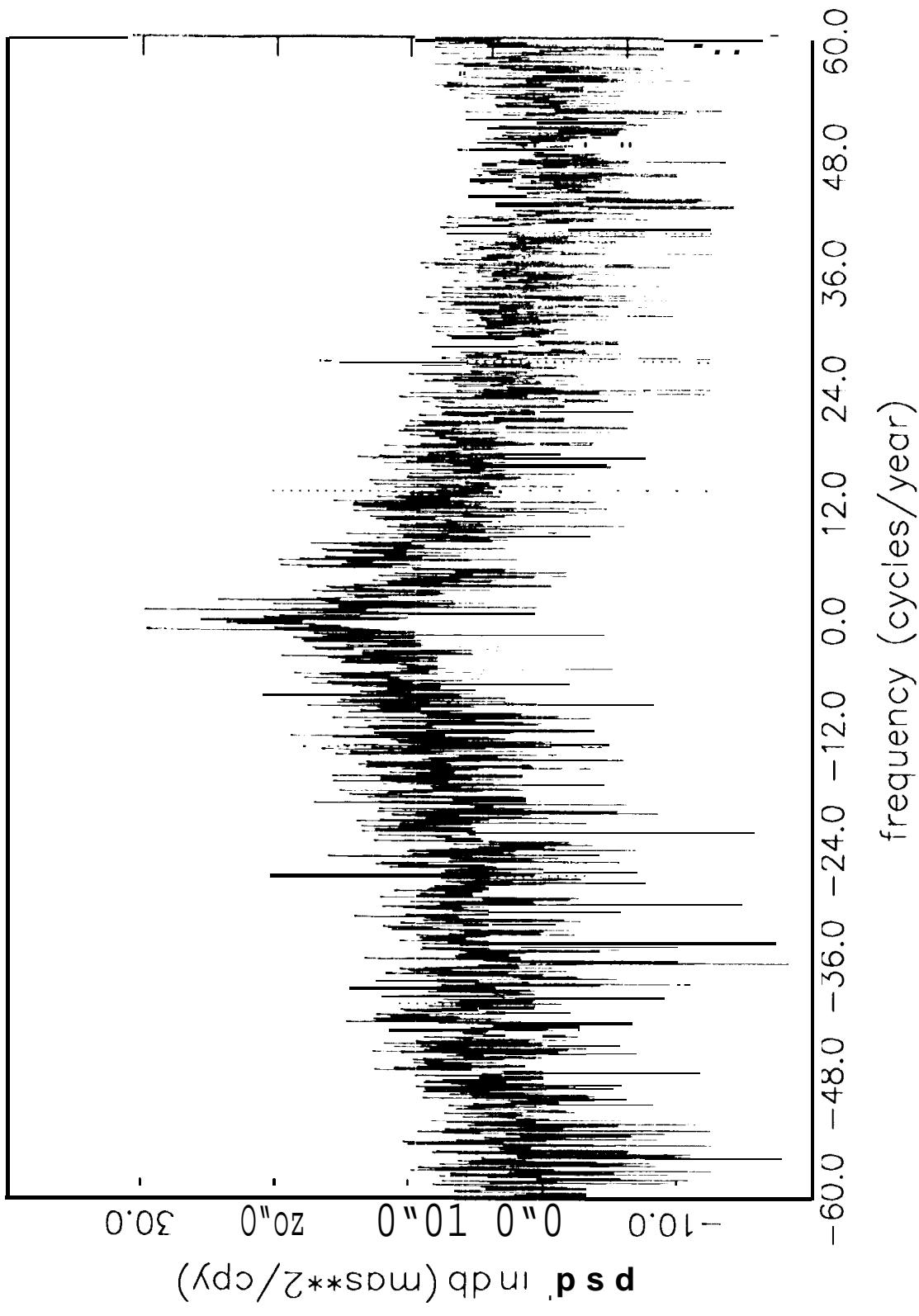
TOTAL WIND + PRESSURE) AAM CHI-FUNCTION



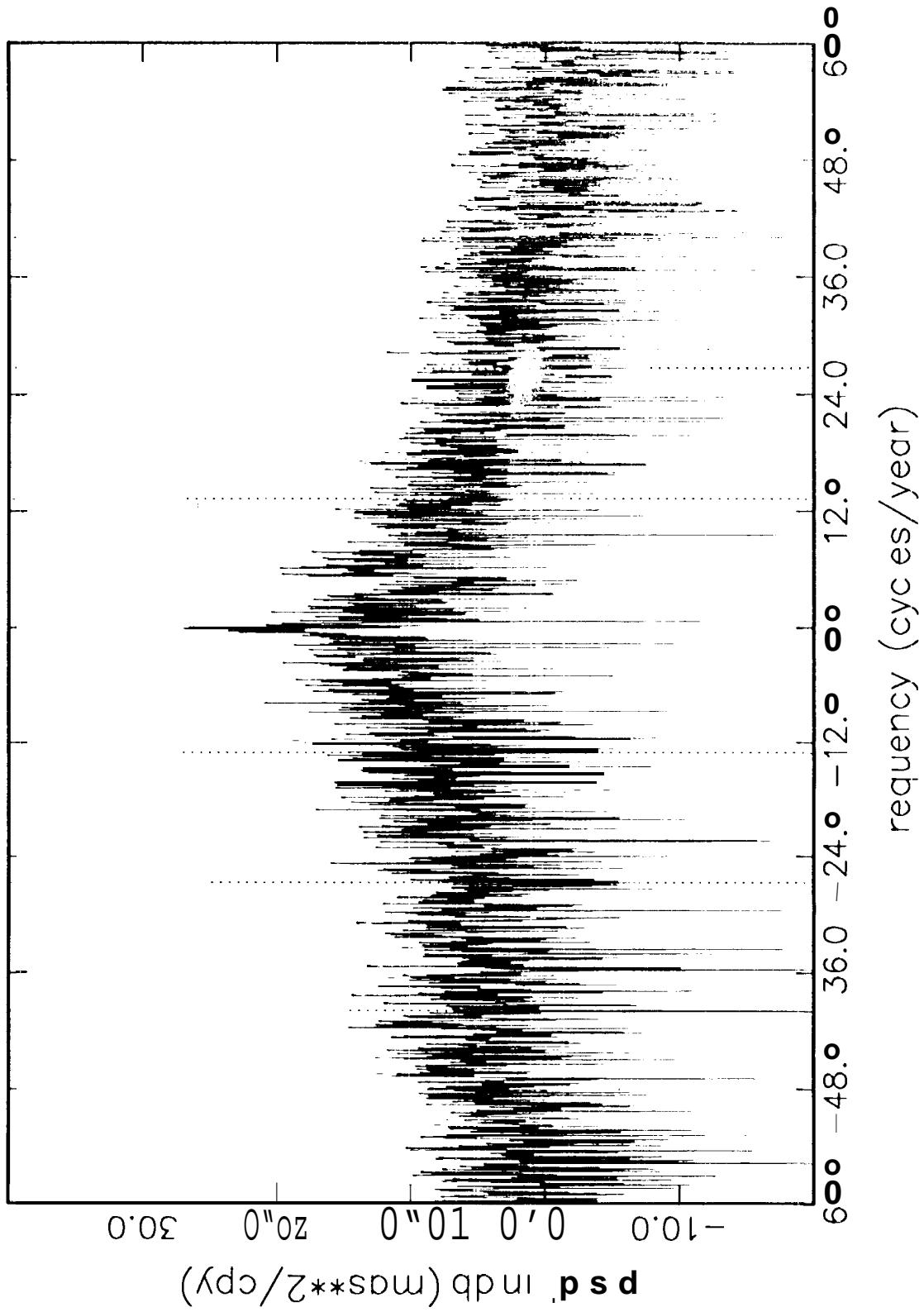
SPACE95 POLAR MOTION CH -FUNCTION 1979–1995)



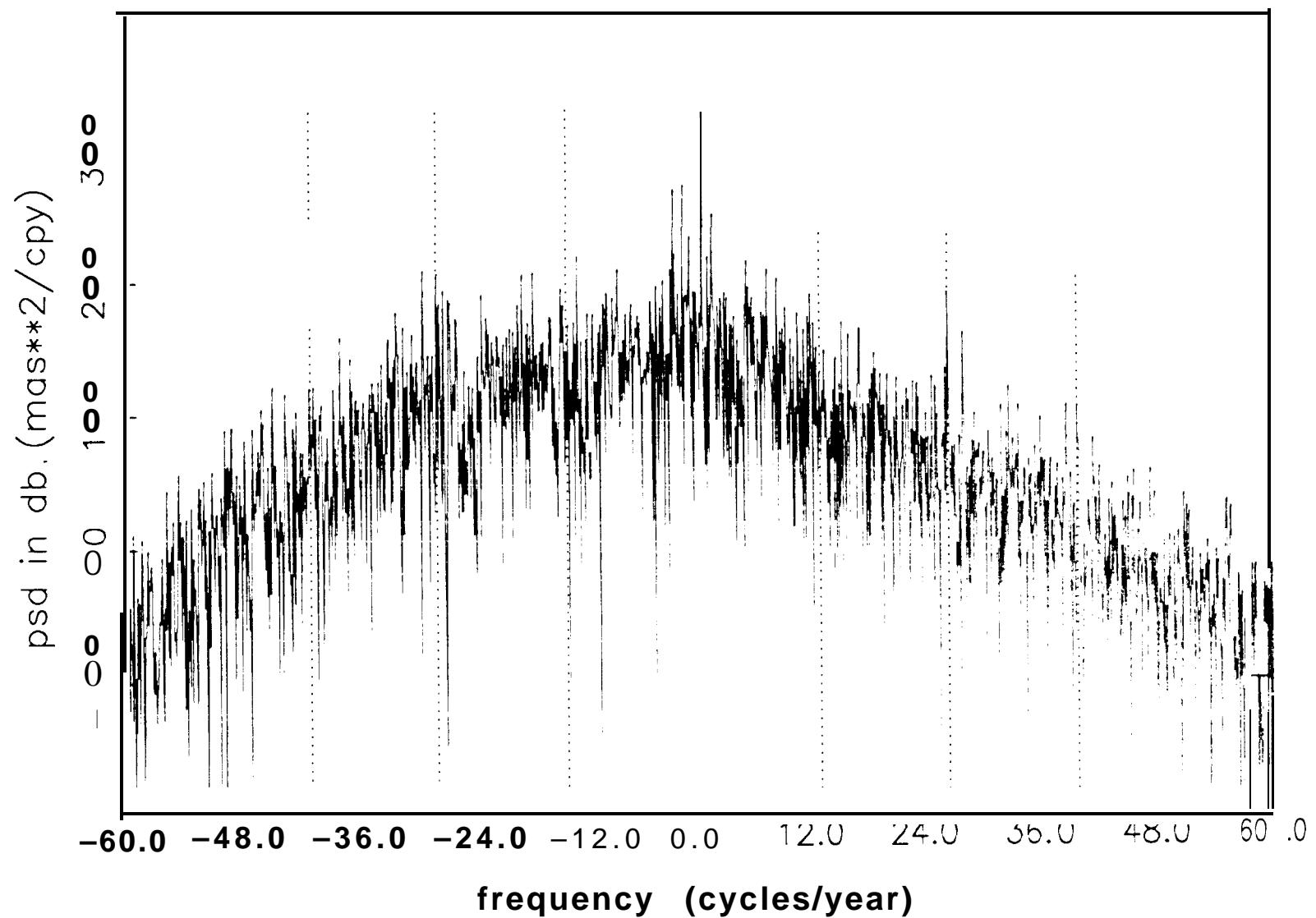
SPAC#95 AAM R&S DUAL CH FUNCTION 1979-1995



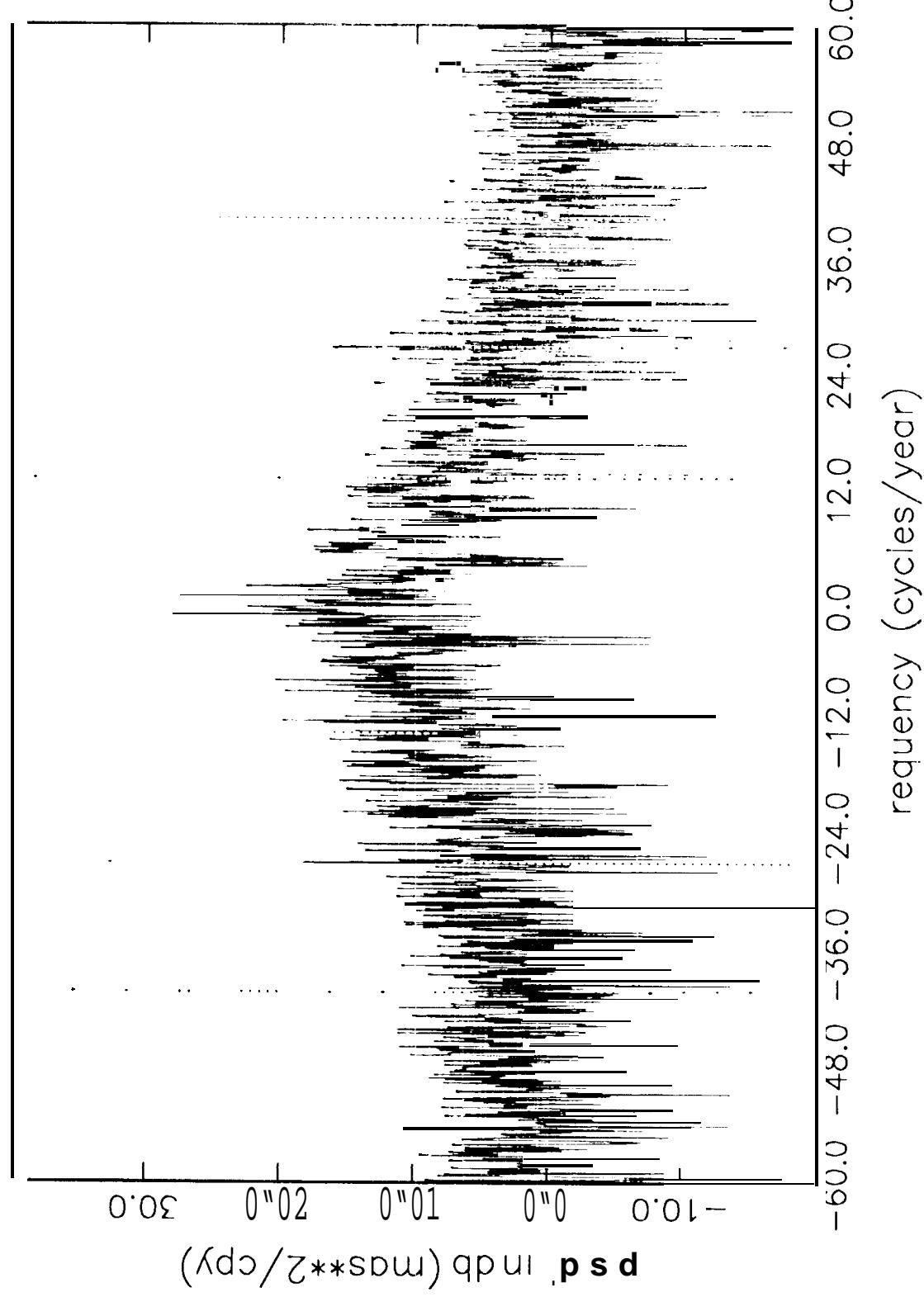
SPACE95-AAM-T DSS RWS,DUAL CHI (1979-1995)



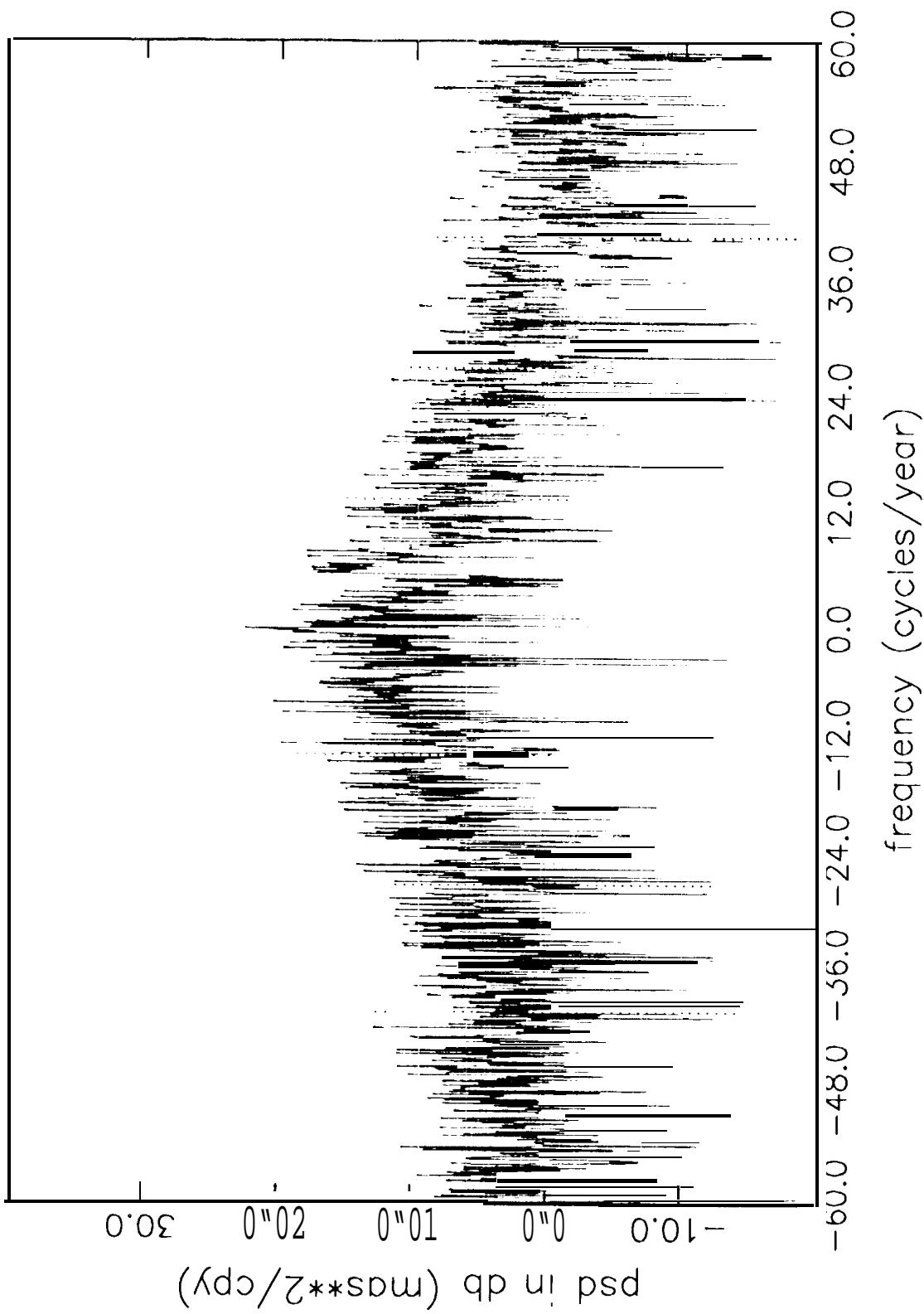
# SPACE95 POLAR MOTION CHI-FUNCTION (1984-1 995)



SPACE95-AAM R<sub>11</sub><sup>11</sup>S DUAL CH -FUNCTION (1984-1995)



SPACW95-AAM-T ~~RES~~ RES DUAL CH 1984-199~~5~~5)



# Observed & Predicted Effects of Long-Period Ocean Tides on the Polar Motion Excitation Function $\chi(t)$

	Prograde Amplitude (mas)	Phase (degrees)	Retrograde Amplitude (mas)	Phase (degrees)
<b><i>M9'</i> (9.12-day)</b>				
SPACE95-AAM	0.68 ± 0.30	-6 ± 25	0.60 ± 0.30	141 ± 28
Gross et al. (1996)	0.54 ± 0.45	38 ± 48	0.21 ± 0.45	79 ± 126
Dickman (1993)	0.13	73	0.21	15
<b><i>M9</i> (.13-day)</b>				
SPACE95-AAM	0.22 ± 0.30	19 ± 78	0.33 ± 0.30	-37 ± 52
Gross et al. (1996)	0.47 ± 0.45	-30 ± 55	0.41 ± 0.45	-95 ± 63
Dickman (1993)	0.32	73	0.52	15
<b><i>Mf'</i> (.3.63-day)</b>				
SPACE95-AAM	1.37 ± 0.30	30 ± 13	1.36 ± 0.30	69 ± 13
Gross et al. (1996)	1.61 ± 0.45	56 ± 16	2.01 ± 0.45	87 ± 13
Dickman (1993)	0.52	100	0.71	8
Seiler/Gross (1993)	0.72	55	0.59	72
<b><i>Mf</i> (13.66-day)</b>				
SPACE95-AAM	1.69 ± 0.30	116 ± 10	2.58 ± 0.30	31 ± 7
Gross et al. (1996)	0.86 ± 0.45	93 ± 30	2.73 ± 0.45	14 ± 10
Dickman (1993)	1.26	100	1.72	8
Seiler/Gross (1993)	1.72	55	1.44	72
Desai (1996)	3.41	158	1.73	-4
<b><i>Mm</i> (27.55-day)</b>				
SPACE95-AAM	0.65 ± 0.30	95 ± 26	0.98 ± 0.30	-55 ± 17
Gross et al. (1996)	0.75 ± 0.45	49 ± 35	0.82 ± 0.45	-59 ± 32
Dickman (1993)	0.47	136	0.28	-7
Seiler/Gross (1993)	0.78	74	0.92	28
Desai (1996)	1.15	-50	0.90	153

Observed values are in blue. Model predictions are in green, red, black.  
 Quoted uncertainties in observed values are 1-sigma formal errors.

Prograde (p) and retrograde (r) amplitudes  $A$  and phases  $a$  defined by:

$$\chi(t) = A_p e^{i\alpha_p} e^{i \phi(t)} + A_r e^{i\alpha_r} e^{-i \phi(t)}$$

where  $\phi(t)$  is the tidal argument.

# RESULTS

- At each tidal frequency, the different observed results, which are based upon different polar motion and AAM data sets spanning different time intervals, agree to within about 1° of each other
- At the  $M_m$ ,  $M_9$ , and  $M_9'$  tidal frequencies, the SPACE95–AAM residual spectrum shows no enhanced power, but the recovered amplitudes are somewhat larger than the formal error
  - Formal error may be underestimated by factor of about 2–3
- Since no resonances in the ocean are expected between the nearby  $M_f$  and  $M_f'$  tidal frequencies, the observed prograde (or retrograde) phases at these frequencies should agree with each other
  - Retrograde phases differ by 38°, or ~1.90
  - Prograde phases differ by 86°, or 3.30
  - Thus, prograde (retrograde) phases agree with each other to within about 3° (or to within about 1 inflated °)
- Model predictions differ with each other, and with observations, by factors as large as 2–3
- Discrepancies between model predictions, and between predictions and observations, illustrate the need for improved models for effect of long-period ocean tides on the Earth's rotation

# **ACKNOWLEDGMENTS**

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